Calculating Arrow Speed from Simple Measurements

by Bertil Olssen

It turns out that using the distance to a sight's aperture from the archer's eye and the distance between the 20 m and 60 m sight marks, arrow speed can be calculated quite accurately. This will come in handy for those of you who do not have access to a chronograph.

Justification for the Calculations

First we need to define some terms: we will let s = distance to target, v_0 = arrow velocity or speed, and α = arrow elevation, that is the angle the arrow makes compared to a horizontal line. We also need the acceleration of gravity which is $g_0 = 9.82$ m/s^2 (at latitude 60-65 degrees). Now, from trigonometry we can state that the distance to the target, s, is:

$$s = [\, 2\, v^2 / g_0 \,] \cdot \sin\alpha \cdot \cos\alpha \quad (\textit{Equation 1})$$

For small angles (that is angles less than about 8°) the cosine of α is very close to 1 thus the arrow velocity for small elevation angles is:

$$v = [\, g_0 \cdot s / 2 \sin\alpha \,]^{1/2} \quad (\textit{Equation 2})$$

a much simpler equation.

Now we define two elevation angles and their difference: α_1 = elevation angle for hitting the target at 20 meters and α_2 = elevation angle for hitting the target at 60 meters and therefore ϕ = the angle between 20 m and 60 m sight marks and the sighting eye (*see diagram*), and therefore:

$$\alpha_2 = \alpha_1 + \phi$$

The arrow velocity (v) is, of course, the same whether we shoot at 20m or 60m, so applying Equation 2 (from above) for the two distances gives us:

$$v = [\, g_0 \cdot 20\text{ m} / 2 \sin\alpha_1 \,]^{1/2} = [\, g_0 \cdot 60\text{ m} / 2 \sin\alpha_2 \,]^{1/2}$$

(*Equation 3*)

Squaring both sides and dividing by $g_0/2$ we get

$$20\text{ m} / \sin\alpha_1 = 60\text{ m} / \sin\alpha_2$$

which reduces to:

$$60\text{ m} \sin\alpha_1 = 20\text{ m} \sin\alpha_2$$

and finally to:

$$\sin\alpha_2 = 3 \sin\alpha_1$$

And then, since $\alpha_2 = \alpha_1 + \phi$:

$$\sin(\alpha_1 + \phi) = 3 \sin\alpha_1$$

For small angles, $\sin(\alpha_1 + \phi) = \sin\alpha_1 + \sin\phi$ giving us:

$$\sin\alpha_2 + \sin\phi = 3 \sin\alpha_1$$

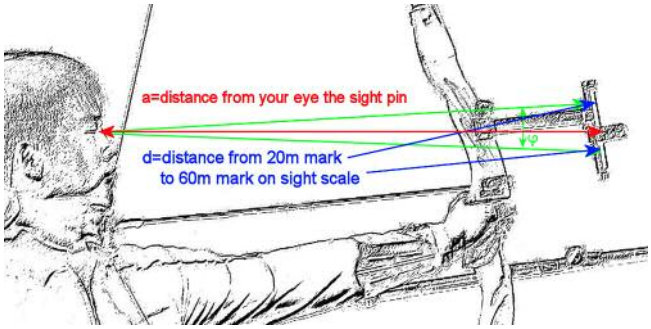
thus: $\sin\alpha_1 = 0.5 \sin\phi$. Substituting $0.5 \sin\phi$ for $\sin\alpha_1$ in Equation 3 and then adding in the value of g_0 and reducing gives us:

$$v = [\, g_0 \cdot 20\text{ m} / 2 \sin\alpha_1 \,]^{1/2} = [\, g_0 \cdot 20\text{ m} / 2 \cdot 0.5 \sin\phi \,]^{1/2}$$

$$\text{or} \quad v = [\, 196.4 / \sin\phi \,]^{1/2} \quad (\textit{Equation 4})$$

Making the Measurements

The only measurements you need to make are the distance from eye to sight pin (a), and the distance between the 20m and 60m sight marks (d) as shown

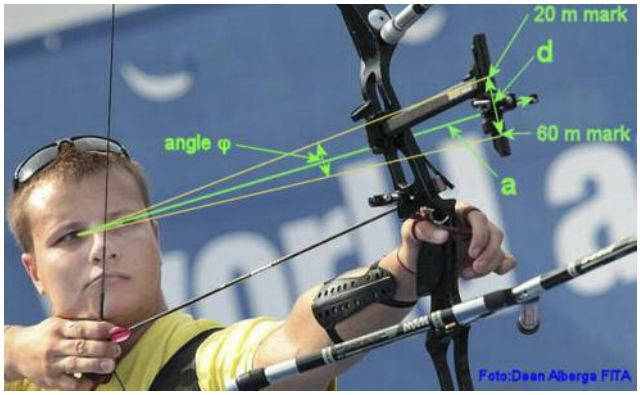


in the above diagram.

By definition, sin ϕ is d/a, and therefore our final equation becomes:

$$v = [\, 196.4 \cdot a / d \,]^{1/2} \quad (\textit{Equation 5})$$

For example, if a = 960 mm and d = 58 mm the equation gives V = 57.0 m/s or 187 ft/s. (If you want your arrow speeds in feet per second all you have to do is multiply the number of meters per second by 3.28.)



Verifying the Formula

To verify this formula actually does work, we set up a chronograph and measured some arrow speeds from a number of different archers and compared the measured speeds with the calculated (*see the blue table*). As you can see there is very little difference between the two.

If you can make the measurements but are not confident making the calculation there is a graph (*see graph next page*) relating the angle ϕ with arrow speed or you can go to my web site as we have set up a calculator for you. All you have to do is enter your measurements (*http://www.ide-teknik.com/arrowspeed.htm*).

Does It Work Also with Compound Bows?

When I submitted this article, your Editor, Steve Ruis, asked me if this formula works with compound bows, so I set about to figure this out. Basically the treatment is the same except that you must adjust for parallax. The angles are measured from the peep sight to the aperture and not from the eye, so to do this you must measure from the peep to the sight's aperture and then from the peep down to the arrow (*see diagram next page*). A little more complicated, but as you can see from the other table (*see yellow table next page*), the results are just as good as for recurve bows.

References

http://margo.student.utwente.nl/sagi/artikel/speed/arrow.html
http://www.bio.vu.nl/thb/users/kooi/tuko92.pdf

Continued on the Next Page

Comparison of Calculated and Measured Arrow Speeds for Recurve Archers

First Name	*Last Name*	*Arrow Type*	*Length (mm)*	*Point (gr.)*	*Vanes*	*Spine*	*Eye to Aptr (mm)*	*20-60 (mm)*	*Speed Calc.*	*Speed Meas.*
Mikael	Ekholm	X10	666	110	Spin-w	550	860	39	65,8	65,3
Göran	Bjerendal	X10	675	100	Spin-w	470	1035	59	63,8	63,2
Hans	Sonesson	ACE	740	90	Spin-w	520	930	46	63,1	63,4
Leif	Jansson	X10	710	100	Spin-w	620	910	52	58,6	
Christine	Bjerendal	X10	637	100	Spin-w	830	860	50	58,1	
Klas	Vängman	ACE	765	95	Arizona	670	1020	60	57,8	
Nore	Åhlund	ACE	707	90	Spin-w	780	900	56	56,2	
Claes	Colmeus	ACE	749	80	K-spin	670	1000	65	55,0	
Bertil	Olsson	ACE	765	70	K-spin	720	1010	66	54,8	55,2
Rolf	Svensson	X10	750	90	Spin-w	600	960	63	54,7	
Ervin	Herbertsson	ACE	706	60	Spin-w	850	753	56	51,4	

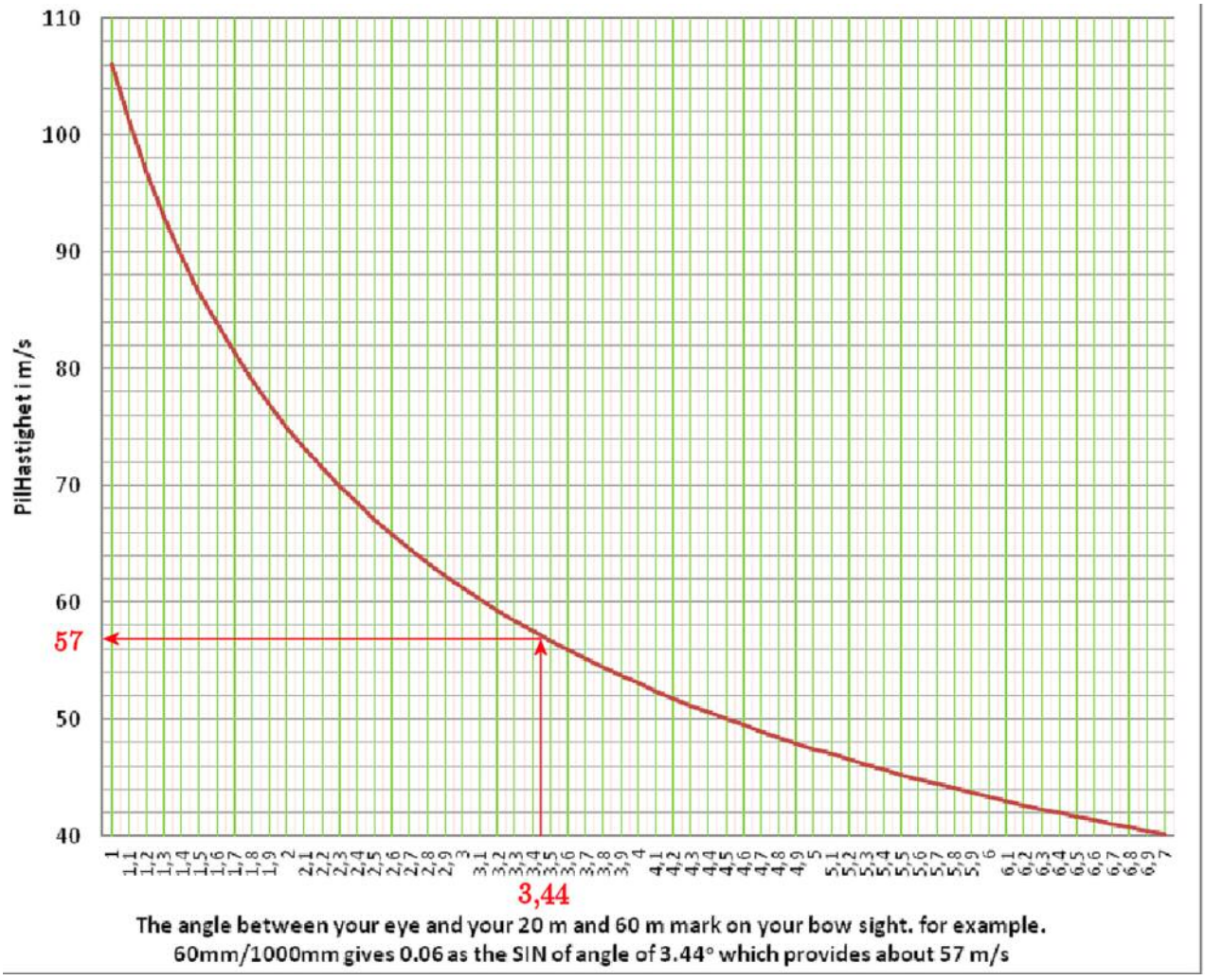
Note: All speeds are in m/s. Also note European usage of comma for decimal point (and vice-versa).

Comparison of Calculated and Measured Arrow Speeds for Compound Archers

First Name	*Last Name*	*Arrow Type*	*Length (mm)*	*Point (gr.)*	*Vanes*	*Spine*	*a_1 & a_2 (mm)*	*d (mm)*	*Speed Calc.*	*Speed Meas.*
Richard	Johnsson	ACE	761	70	Ariz.	500	750/80	19,6	82,6	82,5
Viktor	Lundström	X10	848	120	Ariz.	380	810/75	23,5	78,5	80,1
Olle	Jönsson	ACE	724	95	Ariz.	670	725/83	25,0	72,6	74,2
Håkan	Törnström	SkyA	743	120	Ariz.	400	758/108	19,5	81,8	80,5

Note: All speeds are in m/s. Also note European usage of comma for decimal point (and vice-versa).

Arrow Velocity as a Function of the Angle Between Your Eye and the 20 m and 60 m Marks on your Bow Sight



The angle between your eye and your 20 m and 60 m mark on your bow sight. for example.
60mm/1000mm gives 0.06 as the SIN of angle of 3.44° which provides about 57 m/s